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Protection of a Nuclear Reactor Monitoring System against Gross Measurement Errors

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Abstract

Data Validation and Reconciliation (DVR) is a basic tool for protection of industrial monitoring systems against gross measurement errors. However, the power of this approach is frequently not fully exploited. This paper concentrates on protection of key results against gross and systematic errors.

On-line monitoring can allow a continuous protection of measurement targets against gross errors in measurement chains. The measurement target can be, for example, a nuclear reactor heat output while errors can be hidden in measured flows and state variables of steam and water. The methodology presented here allows a gross error to be identified while maintaining an accurate value of the target variable. In analogy with statistics (power of statistical tests) we can define the probability of success as a power of the Monitoring System Self-Protection (MSSP). A new simple method for assessing the MSSP power is presented.

The MSSP is important for monitoring system maintenance and the optimal placement of additional instruments. It shows which primary variables are self-checked by the redundancy of the system and which should be checked (calibrated) independently. The problem is illustrated by an example taken from one nuclear power station.

Key Words

Data Reconciliation; Gross Errors; Nuclear Reactor Protection

Introduction

In essence, there are at least three major benefits of data reconciliation (DR):

1) Reconciled data are consistent with the model
2) Reconciled data are more precise than data directly measured
3) DR represents a solid basis for detection, identification and elimination of data corrupted by gross errors.

While the first two benefits need not too much discussion, the remaining one deserves a comment. Even if this benefit is often denoted in the literature as "invaluable", the exact knowledge of strength of the DR method is quite scarce. This paper will concentrate on evaluation of the last benefit in practice.

Data Reconciliation

Data Reconciliation (DR) can be defined as an adjustment of measured data to obey some mathematical model (mostly a law of nature). The DR procedure minimizes the generalized sum of squares of adjustments constrained by

$$\boldsymbol{g}(\boldsymbol{z}') = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{z}$ is a vector of process variables (flowrates, temperatures, …) and $\boldsymbol{g}(\boldsymbol{z}')$ is a vector of generally nonlinear functions of $\boldsymbol{z}$. The vector $\boldsymbol{z}$ is partitioned

$$\boldsymbol{z}' = (\boldsymbol{y}', \boldsymbol{x}') \tag{2}$$

where $\boldsymbol{y}'$ is a subvector of unmeasured variables and $\boldsymbol{x}'$ is that of measured variables.

The reconciled solution z' must obey the condition (1) and minimize the generalized sum of squares

$$Q_{min} = \boldsymbol{v}^T\boldsymbol{F}^{-1}\boldsymbol{v} \quad (3)$$

where $\boldsymbol{F}$ is the covariance matrix of measurement errors and $\boldsymbol{v}$ the vector of adjustments of measured variables:

$$\boldsymbol{v} = \boldsymbol{x}' - \boldsymbol{x}^+ \quad (4)$$

where $\boldsymbol{x}'$ are the reconciled values and $\boldsymbol{x}^+$ the vector of measured values subject to random errors.

The solution is based on the assumption that true (unknown) values x are corrupted by random errors e.

$$x^+ = x + e \quad (5)$$

The important notion is the *degree of redundancy* ν. If all unmeasured variables are observable, ν equals the difference between the number of equations and the number of unmeasured variables.

This is a brief statement of the DR problem which has been used in industry since early sixties of the past century. The solution proper was described many times in the literature (for example in books [1,2,3,4,5]) and will not be treated here. Further it is supposed that the reader is acquainted with basics of DR. For those who are not familiar with the DR technology, there is the *Balancing and Data Reconciliation Minibook* [6] available free on the Internet. DR is also mentioned in [9].

Further it is also supposed that there exists a software which is capable of doing all necessary DR activities connected with DR – the DR Engine depicted in the next Figure 1.

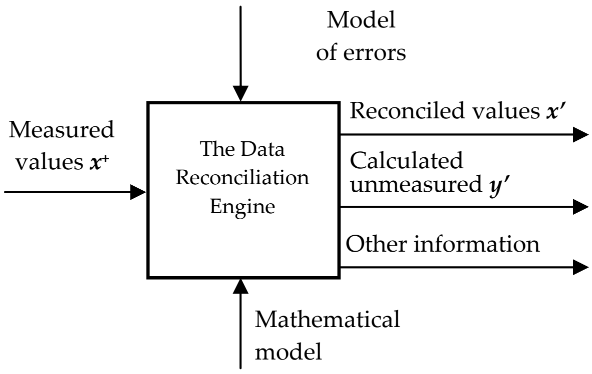

FIG. 1. THE DATA RECONCILIATION ENGINE

We can write symbolically

$$\mathbf{x}' = \mathbf{h}_1(\mathbf{x}^+) \quad (6)$$

$$\mathbf{y}' = \mathbf{h}_2(\mathbf{x}^+) \quad , \quad (7)$$

where $\boldsymbol{h}_1(\boldsymbol{x}^+)$ and $\boldsymbol{h}_2(\boldsymbol{x}^+)$ are functions of measured values. By the *"other information"* in the Fig. 1 we mean other detailed results needed for data analysis described later (mostly covariance matrices of $\boldsymbol{x}'$ and $\boldsymbol{y}'$).

Precision Of Reconciled Data

The precision of data can be characterized by their covariance matrices $\boldsymbol{F}$. Between covariance matrices of $\boldsymbol{x}^+$, $\boldsymbol{x}'$ and $\boldsymbol{v}$ hold the following relation [1]

$$\mathbf{F} = \mathbf{F}_{x'} + \mathbf{F}_{v} \quad (8)$$

The precision of individual variables (elements of vectors) is characterized by their standard deviations σ_i which are square roots of diagonal elements of respective covariance matrices

$$\sigma_i^2 = F_{ii}\,. \quad (9)$$

As $\sigma_{vi}^2 \geq 0$, the following inequality holds

$$\sigma_i \geq \sigma_{x'i} \quad (10)$$

saying that there can be some improvement in precision due to DR. This improvement can be characterized for the i-th variable by the so-called *adjustability* a_i

$$a_i = 1 - \sigma_{x'i}/\sigma_i \tag{11}$$

The adjustability of any measured variable represents the reduction of its imprecision caused by DR. As it will be seen later, adjustabilities are remarkable variables having importance also in area of gross error detection. From the definition follows that any adjustability lies in the interval <0 ;1):

value 0 represents so-called *just determined variable,* which is not influenced by DR and is not adjusted at all (*nonredundant* variable)

value in the interval (0;1) means *redundant variables* which are adjusted in the process of DR

Further it is supposed that covariance matrices of reconciled values $\boldsymbol{x'}$ and of estimated values of unmeasured variables $\boldsymbol{y'}$ are available (the already mentioned DR Engine) and thus providing tolerances (confidence intervals) of reconciled values.

Models

The most important models (laws) applied in DR in the power generation industry are

1) mass balance
2) energy balance
3) momentum balance (flow in pipes and similar tasks)
4) phase equilibrium.

Gross Measurement Errors

Let's modify Equation (5) to the form

$$x^+ = x + e + d \ , \tag{12}$$

where d is a gross error (which is a constant). The most simple and frequently used method for detection of gross errors is the well known chi-square test [1,2,3,4] applied to Q_{min} defined by Eq. (3). Q_{min} has the chi-square distribution with ν degrees of freedom. A gross error is detected when the following inequality holds:

$$Q_{min} > \chi^2_{1-\alpha}(\nu) \tag{13}$$

where $\chi^2_{1-\alpha}(\nu)$ is the critical value of the χ^2 distribution with ν degrees of freedom and the confidence level α (0.05 in our case). The test based on the inequality (13) is called the *Global Test* (GT). If a gross error is detected, the GT is usually followed by the Measurement Test [1,2,3,4] which helps in the location of a gross error source.

Power of the χ2 Test

As every statistical test, also the χ^2 test has its power characteristics shown in Fig. 3.

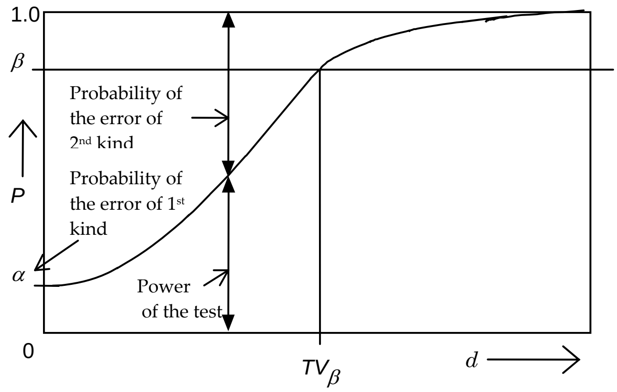

FIG. 3: POWER CHARACTERISTIC OF THE χ^2 TEST

On the *x*-axis there is the magnitude of a gross error. On the *y*-axis is the probability P that the gross error will be detected. The power characteristic for a measured variable equals the confidence level α in the absence of the gross error (d=0) and approaches 1 for high values of the gross error ($d \to \infty$). TV_β is the value of a gross error which will be detected with probability β (β = 0.95 further in this paper). TV_β is characteristic for every measured variable. It is evident that TV_β should be as small as possible.It is clear that gross errors can be detected only for redundant measured variables.

Threshold values can be calculated from equation

$$q_i = \delta_\beta(\nu,\alpha)/[a_i(2-a_i)]^{1/2} \quad (14)$$

where q_i is a dimensionless threshold value TV_i/σ, which means

$$q_i = TV_i/\sigma_i \quad (15)$$

and $\delta_\beta(\nu,\alpha)$ is a constant characteristic for the confidence level of the *chi*-square test α, number of degrees of freedom ν and the probability that a gross error will be detected β.

Equation (14) is slightly re-arranged equation (4.143) from literature [1]. Values of $\delta_\beta(\nu,\alpha)$ are not available in standard statistical tables. Details about calculating threshold values and constants δ (for α=0.05, β=0.9 and ν = 1,2, … ,20) can be found in literature [1]. In this paper we will use the new equation (16) for the more convenient β=0.95. This equation approximates δ (for α=0.05) in the range of ν = 1,2, … ,400). The $\delta_\beta(\nu,\alpha)$ values were calculated with aid of the statistical software [7].

$$\delta_{0.95}(\nu,0.95) = 3.59399 + 0.471951 \ln(\nu) + 0.014197 \ln(\nu)^2 + 0.015074 \ln(\nu)^3 \quad (16)$$

It is worth mentioning that threshold values are simple functions of adjustabilities defined by Eq. (11), see also the next graphical presentation of Eq. (14).

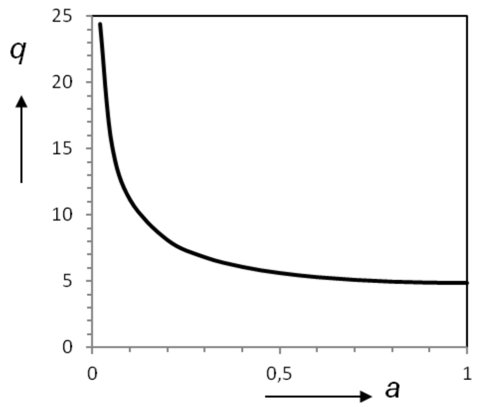

FIG. 4. EXAMPLE OF THE DIMENSIONLESS THRESHOLD VALUE q AS A FUNCTION OF ADJUSTABILITY a (FOR ν= 9, α = 0.05 AND β = 0.95)

Some simple conclusions can be deduced from this graph:

1) the higher is the adjustability, the higher is the probability to detect a gross error (low value of the threshold value)
2) for adjustabilities less than 0.1 the chance for detecting gross errors diminishes steeply

Target Variables and Their Protection against Gross Errors

In practice, there always exist one or several variables, which are of key importance. They are the main reason why hundreds of other variables are measured, collected and processed. The measurement target can be for example a nuclear reactor heat output while errors can be hidden in measured flows and state variables of steam and water. The basic question is: "How are these target variables protected against gross errors of the measurement?"

We are successful if **A**: "A gross error is present and eliminated while maintaining an accurate value for the target variable." We are unsuccessful if **B**: "A gross error is present but not identified and an inaccurate value for the target variable is determined."

In analogy with statistics (power of statistical tests) we can define the probability of an event **A** as a power of the Monitoring System Self-Protection (MSSP).

Let's further suppose that for a target variable *h*, we know (require) the *maximum acceptable error* e_{hmax}. This tolerance can be consumed by

1) a random error e_{hr} caused by random errors of all measured variables (further we suppose Gaussian errors with Normal distribution). As the random errors are not known, we will substitute e_{hr} by e_{hrmax} which represents the tolerance of h caused by random errors (the information provided by the DR Engine).
2) a constant gross error e_{hg} caused by a gross error of one measured variable d in the sense of Eq. (12)

We require that

$$e_{hmax} > e_{hrmax} + e_{hg} \tag{17}$$

Inequality (17) sets the upper limit on the error e_{hg} caused by the gross error, further denoted as e_{hgmax}

$$e_{hgmax} = e_{hmax} - e_{hrmax} \tag{18}$$

This means that both errors' tolerances add to form the overall tolerance. The situation is illustrated in the next Fig. 5.

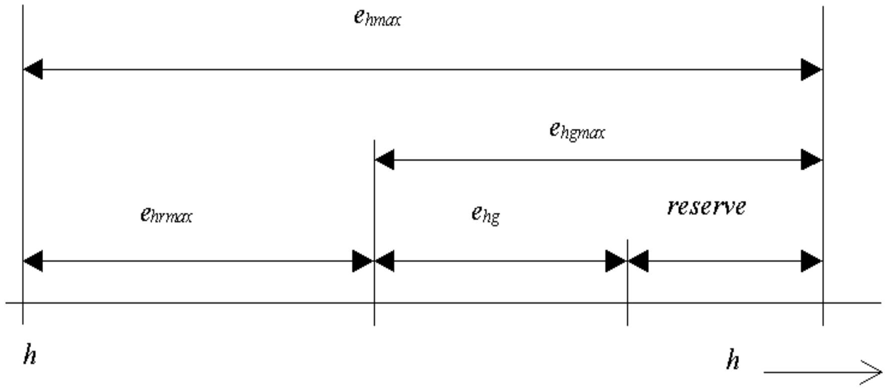

FIG. 5. THE OVERALL TOLERANCE e_{hmax} CONSUMED BY RANDOM AND SYSTEMATIC ERRORS

It is clear that the *reserve* should be non-negative to satisfy our MSSP requirement (17).

The MSSP analysis will be based on a combination of two methods:

1) gross error detection power described in the previous paragraph
2) a parametric sensitivity of the target variable with respect to the individual measured variables.

Let's suppose that a target variable *h* is a function of measured variables in the sense of Eq. (7).

$$h = h(\boldsymbol{x}^+) \tag{19}$$

In this case the function *h()* represents the whole DR process starting by collection of measured values and ending by calculations of target values.

A *parametric sensitivity* ζ_i of *h()* with respect to a measured variable x_i is defined as the partial derivative

$$\zeta_i = \partial h(\boldsymbol{x}^+)/\partial x_i^+ \tag{20}$$

The process consists of two steps, which are applied to all measured adjustable variables:

1) determination of the threshold value for the i-th measured variable
2) evaluation of the parametric sensitivity of the target variable with respect to the i-th measured variable.

The process is illustrated in the next Fig. 6, which is a continuation of Fig. 3. On the right hand side y axis there are errors of the target variable caused by a gross error of the *i*-th adjustable measured variable.

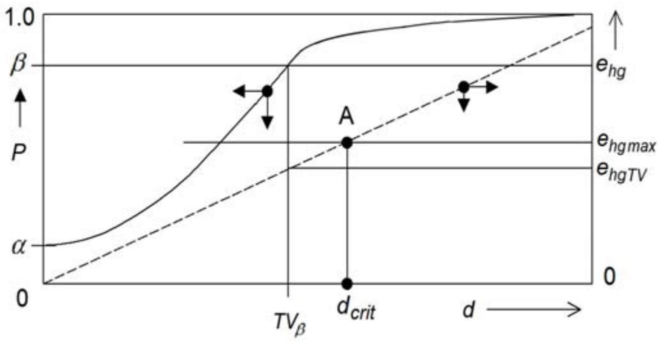

FIG 6: POWER CHARACTERISTICS (FULL CURVE) AND THE PARAMETRIC SENSITIVITY (DASHED STRAIGHT LINE) FOR THE *I*-TH MEASURED VARIABLE (THE INDEX *I* IS OMITTED HERE FOR BREVITY)

It is supposed that the function (19) can be linearised and that a gross error of the *i*-th measured variable transforms to the error of the target variable according to Eq. (19)

$$e_{hg} = \zeta_i d_i \tag{21}$$

This equation is represented by the dashed straight line in Fig. 6. There are two important points on the x axis:

1) threshold value TV_β which informs that gross error was detected (with probability β)
2) critical value of the gross error d_{crit}. At this point e_{hg} reaches the maximum value e_{hgmax} and exhausts all tolerance available (point A in the Fig. 6).

$$e_{hgmax} = |\zeta_i| \, d_{crit,i} \tag{22}$$

or

$$d_{crit,i} = e_{hgmax}/|\zeta_i| \tag{23}$$

Now it is time to compare the power characteristic curve with the parametric sensitivity straight line. The most important is the relation between $d_{crit,i}$ and $TV_{\beta i}$. If there holds the inequality

$$d_{crit,i} > TV_{\beta i} \quad , \tag{24}$$

the gross error will be detected before causing unacceptable error in the target variable and the system is well protected against a gross error of the respective measured variable (this case is depicted in Fig. 6). In the opposite case an undetected gross error can devalue the target value significantly before it is detected. The inequality (24) can be expressed also in the alternative way by substitution of $d_{crit,i}$ from (23) to (24):

$$e_{hgmax} > |\zeta_i| \, TV_{\beta i} \tag{25}$$

saying that

The product of the parametric sensitivity and the threshold value should be less than the tolerance belonging to the gross error set a priori for the target variable.

The inequality (25) thus represents the only criterion for assessing whether the target variable is self protected by DR (and the following data analysis steps) against gross error(s) in the *i*-th measured variable. The inequality (25) must be checked for all measured variables.

Example: Nuclear Reactor Heat Power Monitoring

The heat released in the nuclear reactor is not directly measurable, it is calculated from the mass and heat balance of the feed water and the steam generation systems.

The following example is a simplified version of the 1000 MWe PWR Nuclear Reactor (NR) heat balance problem.

This is a system with 4 steam generators, the auxiliary energy fluxes in the primary circuit (pumps, etc.) are neglected here for simplicity. In Figure 7 there are 3 measurements of condensate flow from the deaerator (streams INPUT) going to the input node (INPUT), 2 measurements of the feed water flow (FWA and FWB) to the feed water header (FWHEAD) and 4 measurements of the feed water to four steam generators SG (FW1, FW2, FW3 and FW4). There are also 4 measured streams of the steam and also purge streams from steam generators and also the measurement of the steam flow leaving the steam header STHEAD. Temperatures are measured for all feed water and steam streams. It is supposed that the steam exiting steam generators is saturated and contains 0.25 % of moisture. The PURGE streams' temperatures are supposed to be equal to the steam temperature leaving the respective steam generator.

The flow of the pressurized water circulating between the reactor and steam generators is not measured, it can be only inferred from the mass and heat balance of steam generators. The hot water circuit balance thus brings no information useful for gross errors detection. The NR heat power is thus determined on the basis of the heat balance of the feed water and steam generators. The heat transfer from the hot water from a nuclear reactor is modelled by 4 (directly unmeasurable) heat fluxes *QSG*. The overall heat flux coming from the reactor *QNR* is calculated as the sum of heat fluxes to the individual steam generators:

$$QNR = QSG1 + QSG2 + QSG3 + QSG4 \tag{26}$$

QNR is the target variable to be determined. The model generates 14 mass and heat balance equations among 28 measured variables and 5 unmeasured variables (heat fluxes QSG and QNR). The mass and enthalpy balance was set up around all nodes excluding the INPUT, where only the mass balance was used, and the NR power defined by Eq. (26). The degree of redundancy is therefore 14 – 5 = 9. There are 9 degrees of redundancy available for DR and gross error detection.

Let's analyse the possibility to protect such system against gross measurement errors. It is required that the overall error of *QNR* should not exceed 1.2 % of the nominal value, which is 3000 MW, i.e. 36 MW.

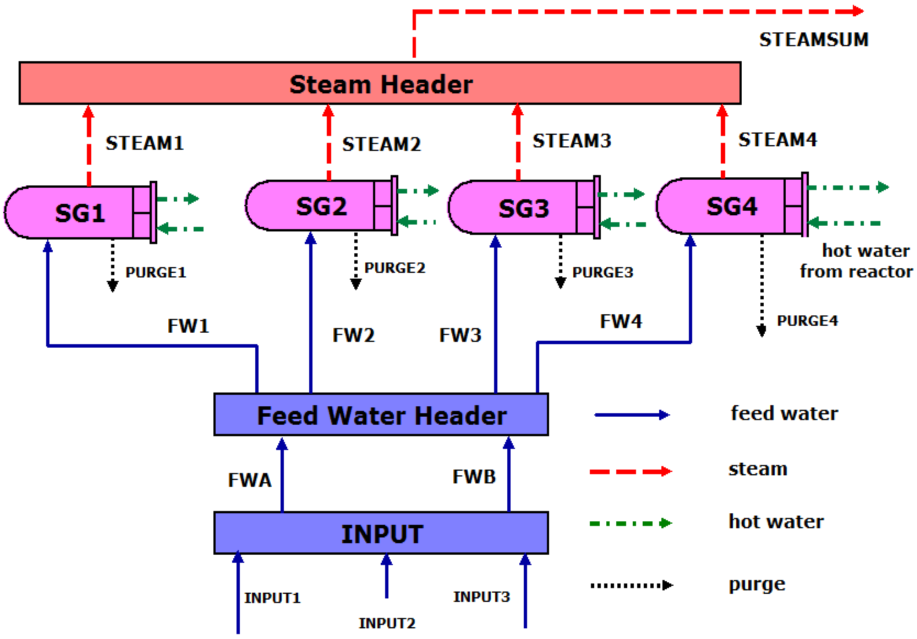

FIG. 7: THE BALANCING FLOWSHEET FOR THE EXAMPLE

Flows and temperatures were measured with tolerances (maximum errors) given in Tab. 1.

The major results of data reconciliation were: Q_{min} = 14.4

(the critical value $\chi^2_{0.95}(9)$ = 16.9, no gross error was detected). The calculated NR heat power *QNR* = 2820.7 ± 10.8 MW, therefore the tolerance of *QNR* belonging to random errors e_{hrmax} equals 10.8 MW (0.38% of the calculated value).

As the maximum allowed tolerance is 36 MW, the undetected gross error should not cause greater error in *QNR* than 36 – 10.8 = 25.2 MW (according to Eq. 18).

Results of the analysis are summarized in the Table 2. As the flowsheet is symmetrical, results will be presented only for the representatives of parallel streams (for example conclusions for all 4 STEAM streams are almost the same).

TABLE 1: TOLERANCES OF MEASUREMENT

Type	Stream	Tolerance
Temperature	All	1 deg C
Flow	STEAM	3%
Flow	PURGE	5%
Flow	INPUT	1.5 %
Flow	FW	1%

TABLE 2: ANALYSIS OF MSSP FOR THE EXAMPLE. TV = THRESHOLD VALUE (THE CRITICAL VALUE OF TV $|\zeta|$ IS 25.2)

Type	Stream	Adjusta-bility *a*	*TV*	Parametric Sensitivity ζ	$TV\,|\zeta|$
Flow	INPUT1-3	0.26	43.3	0.220	9.5
Flow	FW1-4	0.10	21.9	0.989	21.7
Flow	FWA,B	0.21	31.6	0.498	15.7
Flow	PURGE	0.00012	29.1	-1.52	44.2
Flow	STEAM	0.70	30.0	0.110	3.3
Flow	STEAMSUM	0.88	114.8	0.028	3.2
T	FWA,B	0.18	4.3	-1.18	5.1
T	FW1-4	0.042	8.6	-1.17	10.1
T	STEAM	0.026	10.9	0.15	1.6
T	STEAMSUM	0.55	2.8	0.14	0.4
Flow	PURGE*	0.025	2.2	-1.23	2.7

* values after installation of the measurement of the sum of purges

The values in the last column are now compared with the limiting value, which is 25.2 MW according to the Inequality (25). From the Table 2 follows that the target variable *QNR* is quite well protected against gross errors for most of measured variables as they pass the Inequality (25). The only exceptions are the PURGE streams.

Really, any of the purge streams has very low adjustability (and therefore relatively high threshold value) and at the same time also high parametric sensitivity. The value from the last column of Table 2 is 44.2 MW which is almost twice the allowed tolerance for QNR (25.2 MW). This means that the system is not protected against gross errors in purge flow measurements.

Let’s try to raise the redundancy of the instrumentation system. The redundancy of the purge streams is very low (they are checked only by the balance of steam generators, while feed waters and steam has its own redundant balancing sub-flowsheets). By adding the measurement of the sum of all purge streams (tolerance 5 % of the measured value), the problem is completely solved. After this step the threshold values of all purge streams fell from 29.1 to 2.2 kg/s. The result is presented in the last row of Tab. 2. It can be seen that the adjustability of purges increased by more than two orders after the installation of the new measurement.

Interpretation of Results

Results of the Example can be interpreted in the following way. For the whole system we can conclude that it is (after installing the new measurement of the sum of purges) well self-protected against gross errors as concerns the target variable *QNR* and its required tolerance. Especially

The probability that any undetected gross error will impair the required tolerance of QNR (36 MW) is less than 5 %, provided that the measurement of the sum of purges is installed.

Otherwise the flowmeters of purges must be checked independently of the data validation and reconciliation procedure described above. Such interpretation can help in deciding which measured variables are self-protected by DR and which need independent checking, calibration or additional redundancy.

Discussion and Conclusions

Let's briefly discuss some limitations of the proposed method. The solution is based on linearization of the nonlinear model. This is a general problem of the DR technology. It depends on how far from the point of the solution the linearization is applied. In our problem we should look how big the threshold values are, as applied in inequality (25). In practice, if the threshold values are up to 10 % of the flow or up to 10 centigrade in the case of temperatures, the errors introduced by linearization are small and smaller than the other errors (model errors, estimation of measurement precision, etc.). If the threshold values are bigger, it is possible to use the Monte Carlo simulation to check whether the linear model works well.

Conclusions drawn from the method proposed should be applied in the statistical sense. This means that they are valid for a large number of data sets, for example in the case of a continuous monitoring of an industrial process. Benefits of DR are of a statistical nature.

The proposed MSSP analysis is based on the assumption that only a single gross error may exist in the system. This should be the case of a well maintained monitoring system where the probability of multiple gross errors is low. In the case of simultaneous gross errors the problem starts to be more complex (not only for gross error detection but also for their localization).

The method proposed is quite simple and can be useful in the process of analysis of existing monitoring systems. It makes it possible to find which couples of target variables and measured variables are automatically protected against gross error and which primary measurement needs independent checking or frequent calibration. This work can be also useful in the optimization of the instrumentation placement as was shown on example of measurement of the overall purge blowdown.

A NOTE

Example given in this paper was extracted and adapted from a larger problem based on monitoring of a nuclear power plant. There were some simplifications in order to make the problem more tractable in the limited scope of this paper. The Example was solved by the balancing and data reconciliation package RECON. More details about this software can be found on *http://www.chemplant.cz/recon.asp*.

ACKNOWLEDGMENT

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List of Symbols

a adjustability (11)

d gross error (12)

d_{crit} gross error causing error of a target variable equal to e_{hgmax} (23)

e random error with Normal (Gauss) distribution (5)

e_h error of a target variable h

e_{hmax} maximum allowed error of a target variable

e_{hgmax} maximum allowed error of a target variable due to a gross error

e_{hgTV} error of a target variable due to a gross error equal to the threshold value

e_{hrmax} tolerance of error of a target variable due to random errors

e_{max} maximum value of e (1.96σ), tolerance

F covariance matrix

g() column vector of functions (1)

h target variable

q dimensionless gross error (15)

Q_{min} quadratic form of adjustments (3)

$\boldsymbol{v}$ *column* vector of adjustments (4)

$\boldsymbol{x}$ column vector of measured variables

$\boldsymbol{y}$ column vector of unmeasured variables

$\boldsymbol{z}$ column vector of process variables

α level of confidence, probability of the error of 1st kind (0.05 in this paper)

β probability that a gross error will be detected (0.95 in this paper

ν degree of redundancy

χ^2 chi-square distribution

σ standard deviation

Upper index

' reconciled value

$^+$ measured value

$^{-1}$ inverse of a matrix

^{T} transposed matrix (vector)

REFERENCES

[1] Madron F., Process Plant Performance. Measurement and data processing for optimization and retrofits. Ellis Horwood New York, 1992

[2] Romagnoli, J.A., M. C. Sanchez, Data Processing and Reconciliation for Chemical Process Operations, Academic Press London, 2000

[3] Narasimhan, S., C. Jordache: Data Reconciliation & Gross Error Detection. An Intelligent Use of Process Data, Gulf Publishing Company Houston, 2000

[4] Bagajewicz, M.J.: Process Plant Instrumentation: Design and Upgrade. Technomic, Lancaster 2001

[5] Knopf, F.C.: Modeling, Analysis and Optimization of Process and Energy Systems. John Wiley & Sons. Hoboken, NJ 2012

[6] Veverka, V: Balancing and Data reconciliation Minibook, http://www.chemplant.cz/download.asp

[7] The University of Texas, MD Anderson Center, Division of Quantitative Sciences - Department of Biostatistics. The software is available at http://biostatistics.mdanderson.org/SoftwareDownload

[8] RECON - Mass, heat and momentum balancing software with data reconciliation. http://www.chemplant.cz/recon.asp

[9] NUREG/CR-6895: Technical Review of On-Line Monitoring Techniques for Performance Assessment. Division of Engineering Technology, Office of Nuclear Regulatory Research, U.S. Nuclear Regulatory Commission Washington, DC 20555-0001. January 2006